RADIATION BELT DYNAMICS

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Final Report

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14. ABSTRACT
The Earth's inner and outer radiation belts, comprising energetic electrons and protons, pose a hazard to DoD spacecraft. Air Force Research Laboratory (AFRL) has an ongoing research effort to model and forecast the configurations of the belts, and to develop protective technologies for spacecraft. This final report summarizes these developments and research performed at AFRL during FY 2013−2015 to address them, funded by AFOSR grant 13RV08COR, "Radiation Belt Dynamics."

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1. INTRODUCTION

The Earth's inner and outer radiation belts, comprising energetic electrons and protons, pose a hazard to DoD spacecraft. AFRL has an ongoing research effort to model and forecast the configurations of the belts, and to develop protective technologies for spacecraft. This work, recently augmented by observations made by the currently orbiting NASA Van Allen Probes satellites, has revealed unexpected behavior and substantial gaps in our understanding of key processes driving radiation belt dynamics. This final report summarizes these developments and research performed at AFRL during FY 2013–2015 to address them, funded by AFOSR grant 13RV08COR, "Radiation Belt Dynamics."

2. BACKGROUND

The prevailing picture of radiation belt dynamics comprises localized wave-particle interactions, which act as a loss mechanism by scattering particles into the atmosphere, as well as a source by accelerating less energetic (but more numerous) particles. In the long run, these processes are coupled, modified, and brought into steady state by radial transport, most simply described as radial diffusion [1].

There are at least three distinct classes of VLF waves driving inner zone electron radiation belt dynamics: plasmaspheric hiss, lightning-generated VLF, and leakage from terrestrial transmitters. (Another class of waves, known as magnetosonic waves, is also naturally generated and is also receiving increasing attention.) Each propagates at a frequency that can resonate with the cyclotron motion of geomagnetically trapped radiation belt electrons and perturb their orbits, leading to eventual "precipitation" into the atmosphere. The rate for this process is quantified by pitch angle diffusion coefficients and the corresponding timescale, or lifetime, for exponential decay of the particle population.

These three main types of waves, along with Coulomb collisions (but not including radial diffusion, nor energy diffusion) were treated at least in some fashion by Abel and Thorne [2]. They crudely calculated pitch angle diffusion coefficients and the corresponding lifetimes of energetic electrons, which they concluded were roughly consistent with the observed following the 1962 Starfish Prime high-altitude injection. Although this was a landmark study, various pieces of their approach have proven dubious – especially the modeling of VLF transmitter power, which had not yet been checked against measurements.

Each type of wave has its own uncertainties. Perhaps the most fundamental one concerns the level of waves in space originating from the network of large ground-based transmitters operated by the world's major navies. Our early analysis of direct satellite measurements [3] indicated that leakage of these signals through the ionosphere and into space is orders of magnitude (20 dB) below what existing models predicted. This raised the possibility that the ionospheric transmission of lightning-generated whistler waves has also been overestimated, which further suggests that the role of hiss has been underestimated.

Another potentially important process is known as cosmic ray albedo neutron decay (CRAND), by which neutrons are ejected from the atmosphere into space by cosmic rays and spontaneously

decay into charged particles which can become geomagnetically trapped. Since this is a major source of multi-MeV protons in the inner zone and could also be a significant source of electrons, we have carefully reexamined and extended the treatment.

At larger altitude (outside the plasmasphere, at $> 3 - 4$ Re) the dominant resonant waves have similar frequencies but are more variable, driven by magnetospheric activity, and differ in several propagation characteristics. Pitch angle diffusion is inseparable from energy diffusion, and the accompanying cross diffusion is also essential.

Once diffusion coefficients have been formulated and evaluated to describe all these processes, they must be inserted into a corresponding multidimensional diffusion equation. Solving this equation numerically to obtain the actual time-dependent particle behavior is itself a major undertaking.

To address these topics, we have conducted a wide-ranging program as detailed below. Much of the effort was devoted to development and advancement of models, in both the analytical and numerical realms, and this is covered in Section 3. More concrete, specific results of the modeling work are surveyed in Section 4.

3. METHODS, ASSUMPTIONS, AND PROCEDURES

Although energetic particles are the subject of direct interest, evaluating their diffusion coefficients and determining their resulting motion requires a thorough characterization of the relevant wave distributions. Here we review our progress on the major classes of resonant waves, as well as refinements in the evaluation of transport coefficients due to Coulomb collisions and assessment of the CRAND source.

3.1 VLF Transmitters

As stated above, in our earlier work Starks et al. [3] modeled the VLF wave power from ground transmitters present in space, using a 3D ray-and-power tracer combined with long-established "Helliwell curves" for ionospheric absorption [4]. In the case studies performed, in particular of the NPM transmitter observed by the IMAGE satellite, significant disagreement was found. The results have been reconciled by developments on both the data and theory fronts, in close cooperation with collaborators at Stanford University. Subsequently, a much more extensive database of observations by the DEMETER satellite has been compiled, whose statistics yielded electric field measurements systematically larger than the earlier case studies [5]. Conversely, careful reconsideration of the Helliwell calculations revealed that while they are sound in principle, their application is valid only in a restricted set of circumstances. In their stead, a AFRL sponsored the development of a first-principles full-wave propagation code [6, 7] which provides a much more reliable starting point for subsequent ray-and-power tracing. The modeling now reproduces the observations to within a few dB, within the uncertainty of the model inputs, and forms the basis for AFRL's current VLF transmitter modeling under this effort, which feeds into our calculation of diffusion coefficients. We calculate those coefficients individually for about two dozen large transmitters, treated as monochromatic (narrowband)

sources, and globally average the results, accounting for local time and seasonal effects on the ionosphere and plasmaspheric medium.

3.2 Lightning, Hiss, Chorus, and Magnetosonic Waves

By design, ground-based VLF transmitters radiate in a narrow frequency band, so wave-particle interactions are most naturally treated as "coherent." In contrast, naturally occurring waves are generated and interact in an incoherent manner, leading to different mathematical treatment. However, we have shown earlier [8] that both descriptions can be subsumed in a common framework consistent with conventional "quasi-linear" theory. We exploited this treatment to analyze the global dependence and sensitivity of diffusion coefficients to the distributions of frequency and wavenormal angle [9], which are inevitably subject to modeling uncertainties. We further used this analysis to delimit the effects of highly oblique chorus waves on diffusion rates [10]. We similarly characterized the diffusion lifetimes [11], which depend on the rates of diffusion across the entire range of pitch angle, and identified ranges of energy and location which are relatively sensitive or insensitive to details of the wave models.

Since the early estimates of Abel and Thorne, wave models have been in continuous development, driven by both theory and newly available measurements [e.g., 12]. We collaborated in a study [13] using a statistical model of hiss and lightning waves based on direct observations by the CRRES satellite. Colman and Starks [14] developed a model of lightning-generated whistlers based on a global statistical model of lightning strikes, calibrated to direct wave measurements by the DEMETER satellite. In [15], we extended the CRRES-based approach, combining hiss and lightning waves but distinguishing between quiet, moderate, and strong geomagnetic activity. This model was used in a study of observed "peculiar" pitch angle distributions in the inner zone by Albert et al. [16], described in detail in Section 4.

Another class of waves, known as fast magnetosonic waves, is known to be present at some level in the radiation belts, and it has been proposed that they can also have significant effects on energetic electrons [17]. These waves, as recently modeled [17, 18], are also included in our inner zone study [16].

Figure 1 shows the pitch angle, energy, and cross diffusion coefficients at L=2 due to these several types of waves (i.e., combined hiss and lightning-generated whistlers, VLF transmitters, magnetosonic waves) used in our inner zone study [16], as functions of particle electron energy and equatorial pitch angle. The hiss-and-lightning values shown are for moderate activity level (AE), and the transmitter and MS values are for high values of plasmaspheric density, but other versions have also been generated. Line plots at 400 keV are also given for quantitative comparison; it is seen that the timescales are long (tens to hundreds of days), and that even the peak contribution from magnetosonic waves is relatively minor.
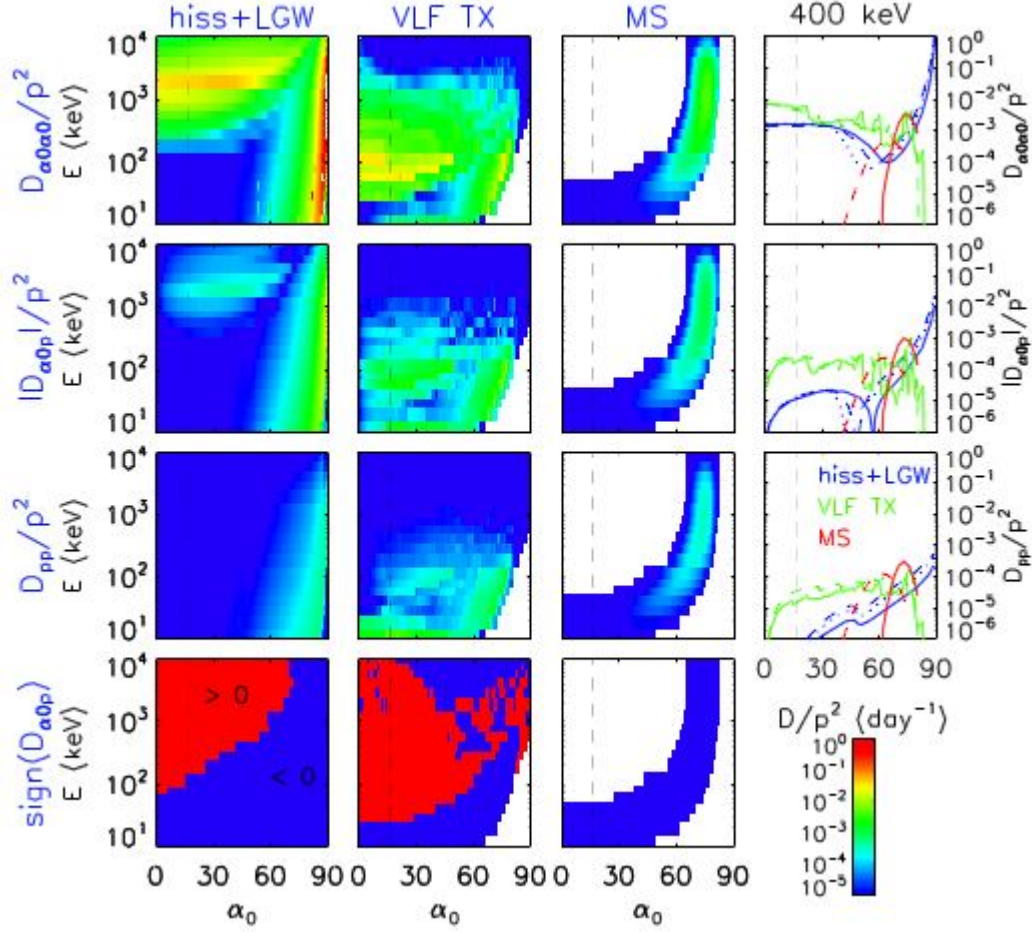
Figure 1. Diffusion Coefficients at L=2 Due to Various Types of Waves

3.3 Coulomb Collisions

We reconsidered the scattering of radiation belt electrons by free electrons, protons, and neutral atoms [19], generalizing the standard treatment to include plasma ions and energy diffusion from range straggling (fluctuations in the energy loss per collision). The resulting transport is usually treated as multidimensional diffusion, but scattering by neutrals was formulated as a generalized Monte Carlo "jump" process to account for rare but important "large angle scattering," which can invalidate the diffusion approximation. The resulting stochastic differential equation was treated with the standard procedure of Monte Carlo simulation backward in time to the initial conditions. In addition to radial diffusion, azimuthal drift was included to account for the distinction between the bounce and drift loss cones at low altitude (i.e., L < 1.5). In further refinements, we replaced all diffusion and drag ("continuous slowing down') approximations with modified stochastic simulation [20]. Also, scattering rates were computed locally in azimuth, rather than drift-averaged. Generally, it was verified that the diffusion approximation is reliable for trapped particles but that the more precise formulations are required to quantitatively model quasi-trapped particles in the drift loss cone. These particles are not very numerous, but can serve as a valuable diagnostic of processes affecting trapped particles.

3.4 Cosmic Ray Albedo Neutron Decay (CRAND)

As mentioned above, a major source of radiation belt protons is CRAND, by which neutrons ejected from the atmosphere by cosmic rays decay into charged particles which become trapped in the geomagnetic field. A complementary source is the direct trapping of energetic protons of solar origin. These sources are usually in balance with loss by the Coulomb collisions, making it difficult to measure them individually, but it has been possible during the recovery following disturbances. We identified such proton observations [21], and found recently refined models to be in good agreement [22]. With this confirmation, we concluded that the assessment of the CRAND contribution to the energetic electron population, believed to be small, is correct [23].

4. RESULTS AND DISCUSSION

After the various physical processes are analyzed and modeled in isolation, as described above, they must be incorporated into global transport codes to simulate actual particle distributions, which can be directly compared to observations. Our developments on this front are described below, for three different classes of phenomena.

4.1 Inner Zone: "Wisp" Feature Associated with the NWC Transmitter

Enhanced levels of several-hundred keV electrons have been observed by the DEMETER satellite, and seemed to be associated with proximity to the powerful NWC (Northwest Cape) VLF transmitter in western Australia [24]. (The enhancement has been described as a "wisp" shape in a two-dimensional plot of flux vs. L and energy.) This provided us a valuable opportunity to test models diffusion and loss by VLF transmitters generally, especially in light of uncertainties concerning transionospheric propagation, as described in Section 3.1.
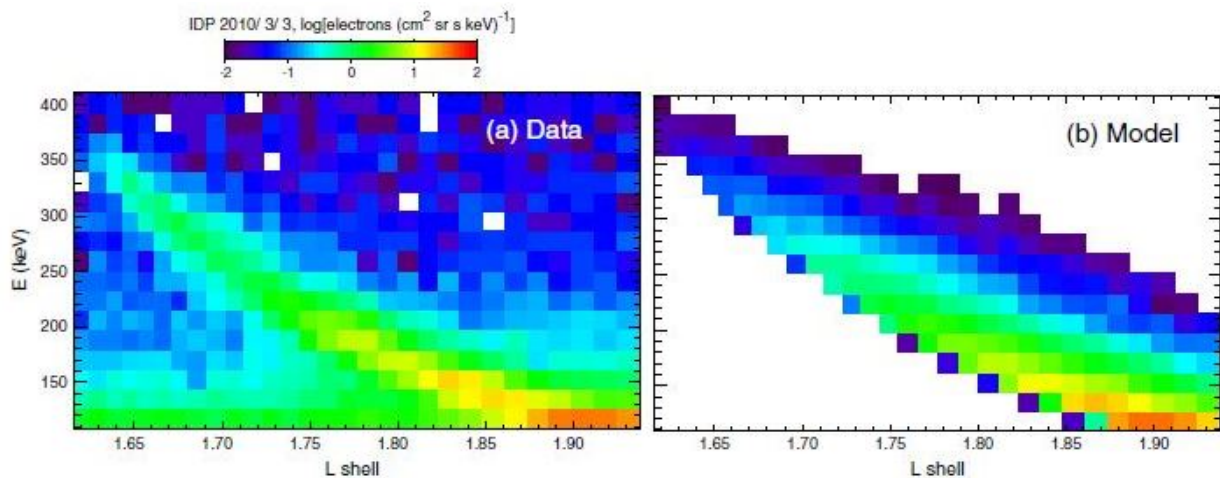

Figure 2. Observed and Simulated Intensity of the Electron "Wisp" Associated With the NWC Transmitter

We developed a simulation incorporating pitch angle diffusion, radial diffusion, azimuthal drift, and Coulomb energy drag, in a stochastic (Monte Carlo) framework [25]. With an outer boundary condition of zero flux in the slot region (L=2.5), and after adjusting the plasmaspheric density, radial diffusion coefficient, and in situ wave power level within realistic ranges, we obtained good qualitative agreement with the observations over a wide radial (L) range, as shown in Figure 2 (reproduced from [25]). The left plot shows the measured electron intensity as a function of L and energy at locations just past (eastward of) the NWC transmitter (at 114 E^o longitude), at local night when ionospheric absorption is at a minimum. The "wisp" feature is of enhanced intensity is clearly seen superimposed on the background. The figure at right shows modeling results, which are in good agreement.

4.2 Inner Zone: "Peculiar" Pitch Angle Distributions

Recent analysis of Van Allen Probes observations has forced a reevaluation of models and our understanding of the inner electron radiation belt. Measurement of MeV electrons is subject to contamination by protons; once this has been corrected for, the electron fluxes are evidently below the level of detectability [26–28]. Furthermore, the pitch angle distributions often show the "peculiar" feature of a local maximum at equatorial pitch angle lower than the expected 90° which corresponds to equatorial mirroring [29, 30]. A sample observation is shown at the left in Figure 3. This is most likely a manifestation of wave-particle interactions, and bears not only on our understanding of the natural environment per se but on the underlying concepts and models.
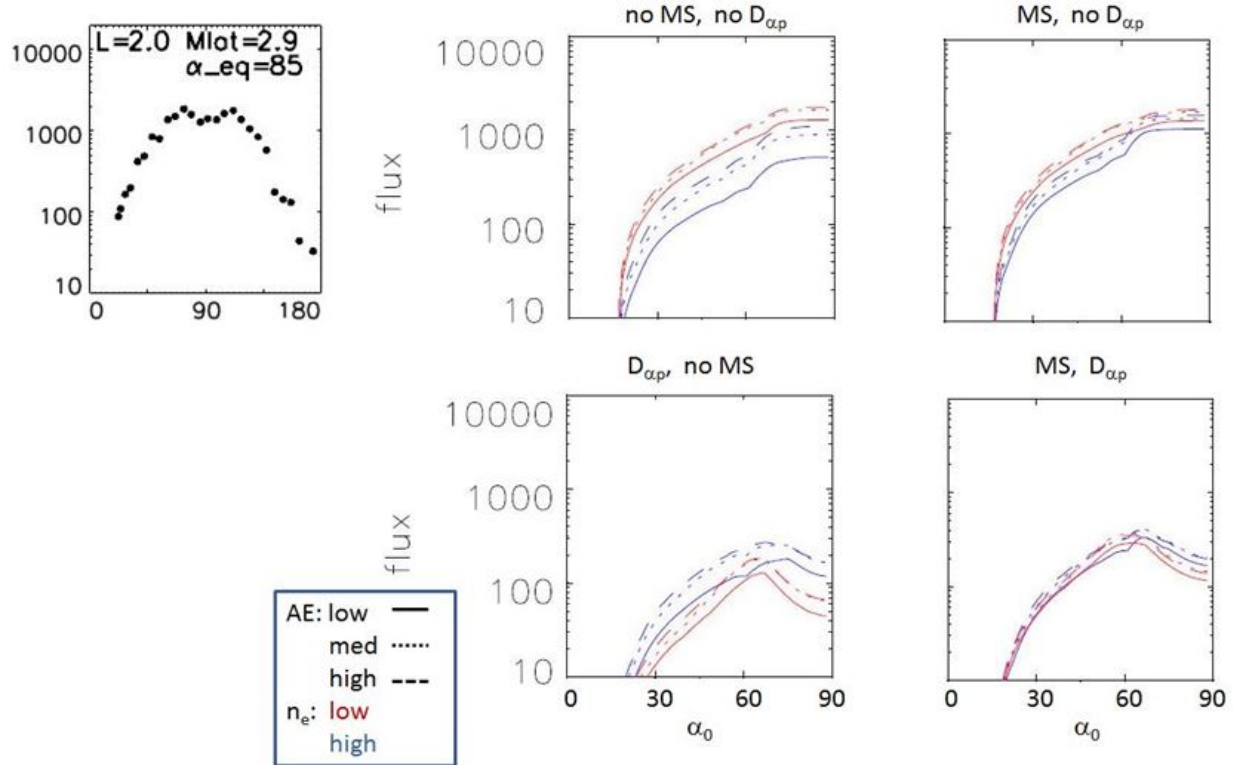.

Figure 3.Observed and Simulated Pitch Angle Distributions of 400 keV Electrons at L=2

We performed a detailed study using the best available models of all relevant waves [16], as discussed in Sections 3.1 and 3.2 and illustrated in Figure 1. Properly accounting for the effects of these multidimensional diffusion coefficients can be difficult, because of numerical issues associated with cross diffusion, but we have developed a technique that transforms away ("diagonalizes") the troublesome terms while still accounting for their effects [31]. We also found it necessary to modify the boundary conditions at 90° appropriately [16]. Applying this procedure to the diffusion coefficients at L=2 led to the steady state pitch angle profiles shown at right in Figure 3. All four plots show results for wave model variants for low, medium, and high magnetic activity (AE), and for low and high plasmaspheric cold electron density (n_e). The upper two of the four plots show results without or with magnetosonic waves, but with cross diffusion omitted in both cases. The lower two plots also show results without or with magnetosonic waves, but with cross diffusion included. It can be seen that for our results to reproduce the observed pitch angle profile, it is necessary to properly include cross diffusion, while magnetosonic waves do not have a major effect. Furthermore, Figure 4 shows that the steady state reached by our simulation leads to an energy distribution that falls off quickly with increasing energy, effectively reproducing the observed 1 MeV threshold
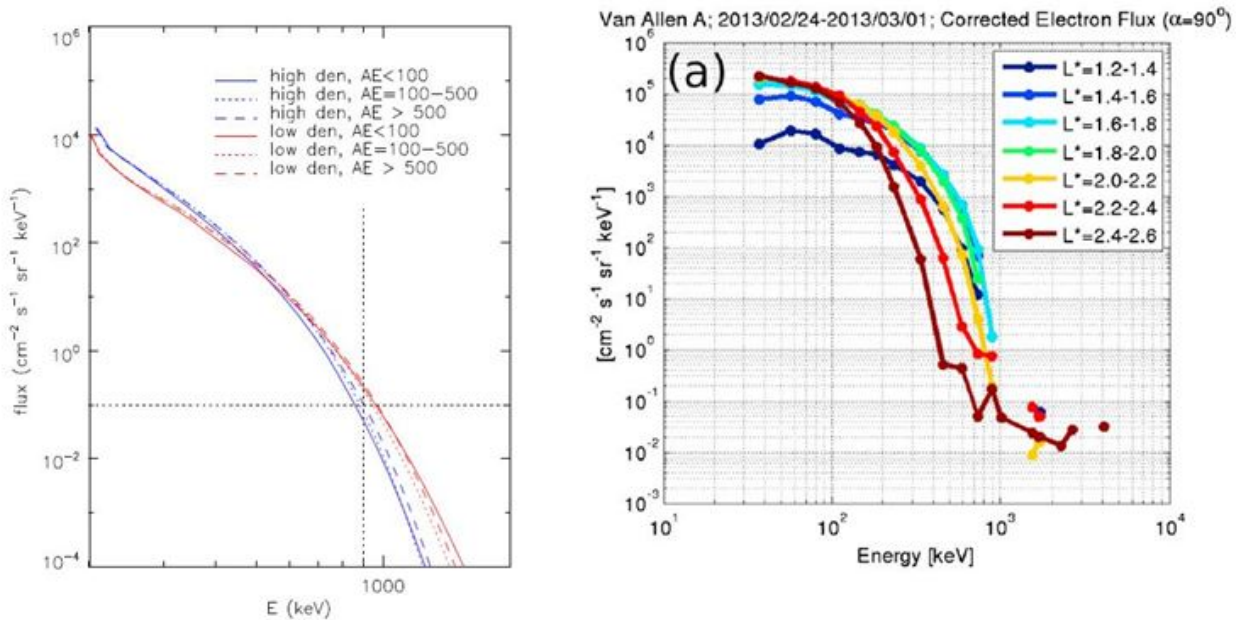
Figure 4. Energy Distributions of Electrons at L=2, Simulated and Observed by VAP.

4.3 Outer Zone

Extensive observations and simulation have established that radial diffusion is insufficient to fully account for electron behavior in the outer radiation belt, during periods of either dropout or recovery, as in our simulations of the 20 September 2007 [32] and October 9, 1990 [33] storms, respectively. Adding the effects of cyclotron-resonant waves, particularly whistler mode chorus, greatly improves the situation, at least during the recovery period following geomagnetic storms, as we found in an improved simulation of the October 9, 1990 storm [34].

Even after the diffusion coefficients for energy, pitch angle, and associated cross diffusion are specified, and combined with radial diffusion, solving the time-dependent multidimensional diffusion equation for the time-dependent particle evolution can present serious numerical difficulties. However, we have largely overcome these by generalizing the previously discussed two-dimensional procedure, which transforms away the cross terms [31], to three dimensions [34]. We have also investigated alternative approaches, including extending stochastic differential equation (SDE, or Monte Carlo) methods [36, 37], and "layer" methods [38, 39], which update values on a grid using SDE concepts via interpolation. Whereas only a few years ago the fully three-dimensional problem was considered prohibitive, we now have several feasible approaches available with different levels of accuracy, convenience, flexibility, and efficiency.

Figure 5, reproduced from [37], shows the results of a simulation, done with collaborators, of a much-studied high speed solar wind stream storm that occurred in 2002. The SDE approach was used to account for diffusion in three dimensions (L, energy, and pitch angle), including cross diffusion between pitch angle and L, due to azimuthal asymmetry ("drift shell splitting") as well as between pitch angle and energy. The relative contributions of radial diffusion and chorus waves, and resulting agreement with observations by GPS satellites, vary with the adiabatic invariants L, M, and K (equivalent to L, energy, and pitch angle), and show where the input wave models are in need of refinement.
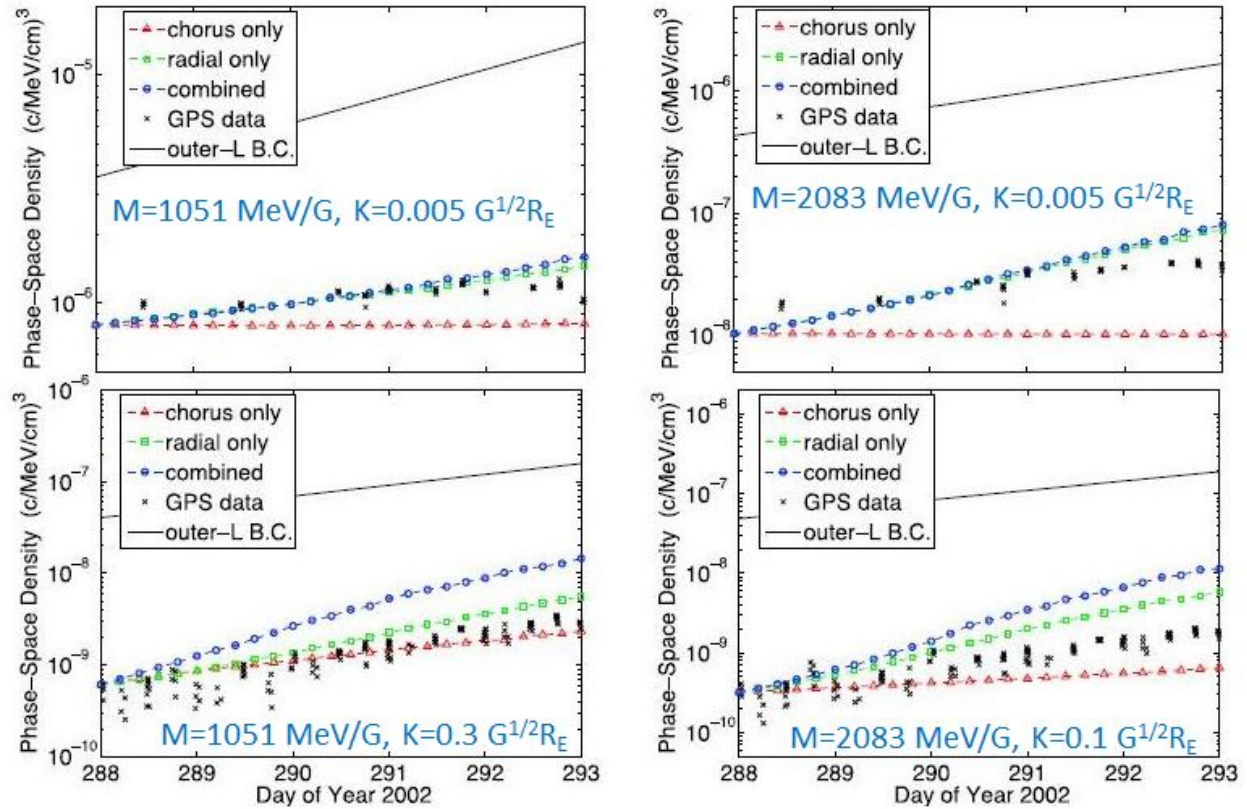

Figure 5. Electron Behavior at L=4 During a Storm in 2002, Simulated and Observed by GPS.

Figure 6, reproduced from [40], shows results from a very large scale simulation, done by collaborators, of the magnetic storm of January 25, 2013. The four-dimensional ring current code RAM-SCB was coupled to the BATS-R-US global MHD code, and augmented with our treatment of time-dependent, local time-resolved pitch angle diffusion. This gave an estimate of the level of electron precipitation into the atmosphere and ionosphere, which affected the ionospheric conductivity, and thus the magnetospheric electric field and global dynamics. The simulated precipitating electron flux is in good qualitative agreement with measurements by the low altitude POES satellites.

These developments will be applied and evaluated in simulation "challenge" studies of four chosen recent events well-observed by the Van Allen Probes satellites, with participation by the radiation belt research community at large [41]. These "GEM challenges" will be coordinated by the Geospace Environment Modeling (GEM) Focus Group "Quantitative Assessment of Radiation Belt Modeling," co-lead by J. Albert (AFRL), W. Li (UCLA), S. Morley (LANL), and W. Tu (UWV).
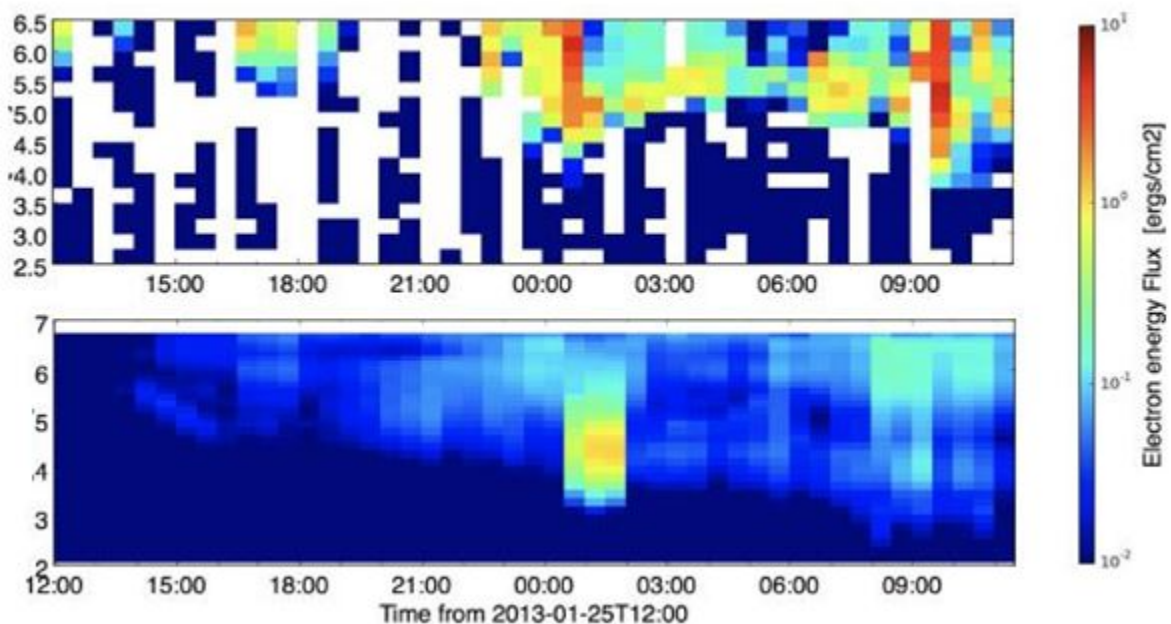
Figure 6. Precipitating Energy Flux, Simulated and Observed by POES.

5. CONCLUSIONS

Because the Earth's inner and outer radiation belts pose a hazard to DoD spacecraft, as noted in the introduction, AFRL has an ongoing research effort to model and forecast the configurations of the belts, and to develop protective technologies for spacecraft. As also noted, both this work and recent observations by the NASA Van Allen Probes satellites have revealed unexpected behavior and substantial gaps in our understanding of key processes driving the belts. One possible approach would be to base specification of the belts on purely statistical analysis of the

database of observations, which we are also pursuing [42]. The approach described here is closer to first principles, with the long-term goal of accurate specification and forecast. This requires both detailed understanding of the underlying microphysical processes and their incorporation into large scale numerical simulations.

Progress achieved under this grant, and papers published or submitted, include:

– Improved understanding and modeling of transionospheric propagation of VLF transmitter power [25]

– The development of wave models for VLF transmitters, hiss, and lightning, as well as magnetosonic waves, and corresponding diffusion coefficients [10, 11, 14]

– Refined treatment of Coulomb collisions, including large-angle scattering into the drift loss cone [19, 20]

– Refined treatment of CRAND for protons and electrons [21, 22, 23]

– Successful modeling of the observe "wisp" feature caused by the NWC transmitter [25]

– Fully two dimensional diffusion simulations reproducing recent observations of unexpected pitch angle and energy distributions of inner zone electrons [16, 27]

– The development of several numerical algorithms (diagonalization, SDE, layer) for multidimensional diffusion simulations of electrons in the outer zone [35, 37, 39]

– Preliminary application of diffusion processes in very large scale simulations of the entire magnetosphere [40]

Our recently improved understanding of the inner zone now seems adequate to explain most current observations, and to proceed with the investigation of improved spacecraft technologies. The outer zone is more complex and dynamic, but comprehensive, large scale simulations are getting to the point of including almost all known key microphysics, and continue to advance rapidly.

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LIST OF SYMBOLS, ABBREVIATIONS, AND ACRONYMS

AE   auroral electrojet index

AFOSR   Air Force Office of Scientific Research

AFRL   Air Force Research Laboratory

BATS-R-US   Block-Adaptive Tree Solar-wind Roe Upwind Scheme

CRAND   Cosmic Ray Albedo Neutron Decay

dB   decibel

DoD   Department of Defense

GPS   Global Positioning System

keV   kiloelectron Volt

L   a measure of distance from Earth, in units of Earth radius

MeV   Megaelectron Volt

MHD   magnetohydrodynamic

MS   magnetosonic

NWC   Northwest Cape transmitter

PA   pitch angle

POES   Polar-orbiting Operational Environmental Satellites

SDE   stochastic differential equation

TX   transmitter

VAP   Van Allen Probes satellites

VLF   Very Low Frequency

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